# APPENDIX I

## NOTATIONS AND SYMBOLS

This appendix contains a summary of notations used in this book. Definitions of the quantities encountered here can be found either in the following Appendix, or in the proper parts of the text where they are first introduced.

### I.1 Summation Convention

Einstein convention:

(I.1-1) $$\sum_i A_i B^i \equiv A_i B^i$$

but $A_i B_i$ indicates no summation.

### I.2 Vectors and Vector lndices

(a) Vectors in Euclidean spaces are denoted by boldface single Latin letters, e.g. $x$, $y$,... etc.

Unit vectors are denoted by an overhead caret. e.g. $\hat{\boldsymbol{e}}$, $\hat{\boldsymbol{u}}$, $\hat{\boldsymbol{z}}$... etc.

Orthonormal basis vectors are denoted by $\left\{\hat{\boldsymbol{e}}_i\,,\; i = 1, 2, \ldots, n\right\}$ so that

(I.2-1) $$\boldsymbol{x} = \hat{e}_i\, x^i$$

(b) For spaces with 'metric tensors', say $g_{i\,j}$, the covariant $x_i$ and contravariant $x^j$ components of the same vector is related by

(I.2-2) $$x_i = g_{i\,j}\, x^j$$

such that $x_i\, y^i$ is an invariant. The metric tensor for Euclidean spaces is the Kronecker delta function: $g_{i\,j} = \delta_{i\,j}$. Hence, for Euclidean spaces, $x_i = x^i$.

(c) Vectors in general linear vector spaces are denoted by kets

$$|\, x\, \rangle,\ |\, \xi\, \rangle,\ \ldots$$

or bras

$$\langle\, f\, |,\ \langle\, \psi\, |,\ \ldots$$

Whenever confusion is not a danger, we shall omit the bra and ket signs, and use bold letters to denote the corresponding (general) vectors, as for ordinary vectors.

(d) Multiplication of a vector $|\, x\, \rangle$ by a number $\alpha$ can be written in three equivalent ways:

(I.2-3) $$|\, \alpha\, x\, \rangle = \alpha\, |\, x\, \rangle = |\, x\, \rangle\, \alpha$$

(e) Lower indices are used to label 'ket' basis vectors, e.g.

$$\{\, |\, e_i\, \rangle\, ;\ i = 1, \ldots n\, \}$$

Upper indices are used to label components of ket-vectors.

Consequently, if $x^i$ are components of $|\, x\, \rangle$ with respect to $\{\, |\, e_i\, \rangle\, \}$, then

(I.2-4) $$|\, x\, \rangle = \sum_{i=1}^{n} |\, e_i\, \rangle\, x^i = |\, e_i\, \rangle\, x^i$$

(f) Upper indices are used to label basis-vectors of the dual space, e.g.

$$\left\{\left\langle\, e^i\, \right|;\ i = 1, \ldots n\right\}$$

Lower indices are used to label components of bra-vectors, i.e.

(I.2-5) $$\langle\, x\, | = \sum_{i=1}^{n} x_i \left\langle\, e^i\, \right| = x_i \left\langle\, e^i\, \right|$$

If $\langle x |$ and $\langle e^i |$ are dual to $| x \rangle$, $| e_i \rangle$ respectively, then $x_i^{\dagger} = x^{i*}$ where $^*$ indicates complex-conjugation.
The raising and lowering of the index in this way is a desirable convention, since the scalar product can be written as

(I. 2-6) $$\langle x | y \rangle = \sum_{i=1}^{n} x_i^{\dagger} y^i = x_i^{\dagger} y^i$$

### I.3 Matrix Indices

As usual, elements of a matrix will be labelled by a row index followed by a column index. (In rare occasions, each index may consist of more than one symbols.) The transpose of a matrix, indicated by the superscript *T*, implies the interchange of the row and column indices. We write,

(I.3-1) $$A^T{}_{ij} = A_{ji} \qquad A^T{}_i{}^j = A^j{}_i$$

Note that superscripts (subscripts) remain as superscripts (subscripts): they are not affected by transposition.

Because of the Einstein summation convention adopted above, both upper and lower indices are used to label matrix elements. The normal notation for matrix multiplication is

(1.3-2) $$(A\,B\,C)^i{}_j = A^i{}_k\, B^k{}_m\, C^m{}_j$$

which naturally follows from the convention adopted above for vectors--as is shown in Sec. II.3.

Just as in the case of vector components, it is desirable to switch upper and lower indices of a matrix when its complex conjugate is taken. As hermitian conlugation also implies taking the transpose, it is natural to incorporate also (I.3-1), and arrive at the convention:

(I.3-3) $$A^{\dagger\, i}{}_j = A^{*}{}_j{}^i = \left(A^j{}_i\right)^{*}$$

If *A* carries any other indices (such as a representation label), we shall raise or lower them according to the same convention.

As indices may also be raised or lowered by contraction with a metric tensor, as mentioned in Sec. I.2, variants of Eq. (I.3-2) may look like:

(I.3-4) $$\begin{aligned}(A\;B\,C)^i{}_j &= A^{ik}\, B_{km}\, C^m{}_j \\ &= A^i{}_k\, B^{km}\, C_{mj}\end{aligned}$$

All these forms are equivalent if the raising and lowering of indices are carried out consistently.