5.4. Quantum Statistical Physics: Fixed Number of Particles

Consider a system of n identical non-relativistic particles with

$$H_n = \sum_{i=1}^{n} \frac{1}{2\,m} \hat{p}_i^2 + V(\hat{q}_1,\ \ldots,\ \hat{q}_n) \tag{5.79}$$

where

$$V(\ldots \hat{q}_i,\ \ldots,\ \hat{q}_j,\ \ldots) = V(\ldots \hat{q}_j,\ \ldots,\ \hat{q}_i,\ \ldots) \qquad \forall\ i,\ j$$

The statistical properties of the particles are expressed as in the total symmetrization (anti-symmetrization) of the wave function for bosons (fermions).

Formally, one can still write

$$\mathcal{Z}(n,\ \beta) = \int [d\,q_i(t)]\, e^{-S(q)/\hbar} \tag{5.80}$$

with

$$S(q) = \int_0^\beta d\,t \left\{ \sum_i \frac{1}{2} m\,\dot{q}_i^2 + V[q(t)] \right\} \tag{5.81}$$

$$= \hbar \int_0^\beta d\,\tau \left\{ \sum_i \frac{1}{2\,\hbar} m q_i'^{\,2} + V[q(t)] \right\}$$

where

$$\tau = \frac{t}{\hbar} \qquad\qquad q_i' = \frac{d\,q_i}{d\,\tau}$$

However, since $\mathcal{Z}(n,\ \beta)$ is a trace over the properly symmetrized n-particle Hilbert space, the boundary conditions should be accordingly permutated. This makes the formulism rather complicated & we shall describe in the next section a much less involved formulism that deals with systems with varying number of particles.