5.2.d. Spin of Photons

Consider the Coulomb gauge (see 5.2.c._CanonicalQuantization_LinearMediumCoulombGauge.pdf).

The EM field is a sum of 2 independent K-G fields specified by the polarization vectors $\boldsymbol{\varepsilon}^{\lambda}(\boldsymbol{k})$ with $\boldsymbol{k}\cdot\boldsymbol{\varepsilon}^{\lambda}(\boldsymbol{k})=0$.
The quanta of these fields are called photons.

One can rotate these vectors around $\boldsymbol{k}$ by an angle θ to get

$$\boldsymbol{\varepsilon}_{\theta}^{1}=e^{i\theta J_{//}/\hbar}\,\boldsymbol{\varepsilon}^{1}=\boldsymbol{\varepsilon}^{1}\cos\theta+\boldsymbol{\varepsilon}^{2}\sin\theta$$

$$\boldsymbol{\varepsilon}_{\theta}^{2}=e^{i\theta J_{//}/\hbar}\,\boldsymbol{\varepsilon}^{2}=-\boldsymbol{\varepsilon}^{1}\sin\theta+\boldsymbol{\varepsilon}^{2}\cos\theta$$

where $J_{//}=\boldsymbol{J}\cdot\hat{\boldsymbol{k}}$ is the component of the internal angular momentum (spin) $\boldsymbol{J}$ in the direction of $\boldsymbol{k}$.

Eigenstates of $e^{i\theta J_{//}/\hbar}$ are

$$\boldsymbol{\varepsilon}^{\pm}=\frac{1}{2}\left(\boldsymbol{\varepsilon}^{1}\mp i\,\boldsymbol{\varepsilon}^{2}\right)$$

since

$$\begin{aligned}e^{i\theta J_{//}/\hbar}\,\boldsymbol{\varepsilon}^{\pm}&=\frac{1}{2}\left[\boldsymbol{\varepsilon}^{1}\cos\theta+\boldsymbol{\varepsilon}^{2}\sin\theta\mp i\left(-\boldsymbol{\varepsilon}^{1}\sin\theta+\boldsymbol{\varepsilon}^{2}\cos\theta\right)\right]\\&=\frac{1}{2}\left[\boldsymbol{\varepsilon}^{1}(\cos\theta\pm i\sin\theta)\mp i\,\boldsymbol{\varepsilon}^{2}(\cos\theta\pm i\sin\theta)\right]\\&=e^{\pm i\theta}\,\boldsymbol{\varepsilon}^{\pm}\end{aligned}$$

For $\theta<<1$,

$$(1+i\theta J_{//}/\hbar+\ldots)\,\boldsymbol{\varepsilon}^{\pm}=(1\pm i\theta+\ldots)\,\boldsymbol{\varepsilon}^{\pm}$$

→ $$J_{//}\,\boldsymbol{\varepsilon}^{\pm}=\pm\hbar\,\boldsymbol{\varepsilon}^{\pm}$$

i.e., $\boldsymbol{\varepsilon}^{\pm}$ are also the eigenstates of $J_{//}$ with ± helicity.