7.1. Topological Sectors

Quantization of fluctuation around classical vacuum $\phi(\boldsymbol{x}) = v$ → point particles.

Classical nonlinear field eq. → Solitons (extended particle-like object)
Quantization → Coherent states of collective excitations

Topological soliton: stability guaranteed by topology.
Eg.: Vortices, Skyrmions, sine-Gordon solitons, ...

For an N-component field $\phi(\boldsymbol{x})$ on m-D space,

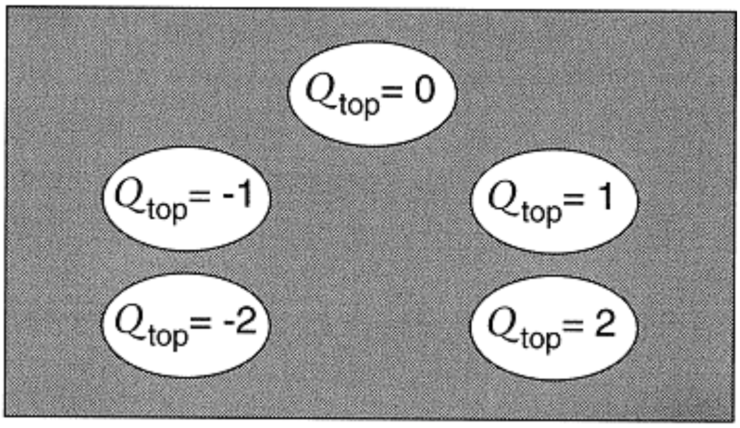
Field manifold = Set of all mappings $R^m \to C^N$.

Energy of field $E(\phi) = H$.

Consider submanifold with $E(\phi)$ finite.
Note: $E(\phi) < \infty$ imposes B.C. on $\lim_{|\boldsymbol{x}|\to\infty} \phi(\boldsymbol{x})$.

All $\phi(\boldsymbol{x})$ that is continuously deformable (homotopic) to classical vacuum $\phi(\boldsymbol{x}) = v$ without violating E-finiteness belongs to the $Q_{top} = 0$ sector.

For example, a solution of the Euler eq, although in a local minimum of H, is not stable if it is homotopic to the classical vacuum. In which case, it may tunnel to the latter. If the tunneling process is prohibited by energy-momentum conservation, the solution is called a non-topological soliton.



Disjoint sectors of E-finite submanifold labeled by the topological charge Q_{top}.

Different sectors are separated by infinite energy barriers.

Existence of topological soliton depends little on H.
The same topological solitons can appear in systems with very different H.