7.7.c. Examples

Skyrmion

The simplest analytic function is

$$\omega(z) = \left(\frac{z}{\kappa}\right)^n = \left(\frac{\rho}{\kappa}\right)^n e^{i n \theta} \qquad \text{where} \qquad z = \rho\, e^{i\theta}$$

$$\omega(z)^* = \left(\frac{z^*}{\kappa}\right)^n = \left(\frac{\rho}{\kappa}\right)^n e^{-i n \theta}$$

$$\to \qquad \phi_3 = \frac{|\,\omega\,|^2 - 1}{|\,\omega\,|^2 + 1} = \frac{\rho^{2n} - \kappa^{2n}}{\rho^{2n} + \kappa^{2n}}$$

$$\phi_1 = \frac{\omega + \omega^*}{|\,\omega\,|^2 + 1} = \frac{2\left(\frac{\rho}{\kappa}\right)^n \cos n\theta}{\frac{\rho^{2n}}{\kappa^{2n}} + 1} = \frac{2\,\rho^n \kappa^n}{\rho^{2n} + \kappa^{2n}} \cos n\theta$$

$$\phi_2 = i\,\frac{\omega - \omega^*}{|\,\omega\,|^2 + 1} = -\frac{2\left(\frac{\rho}{\kappa}\right)^n \sin n\theta}{\frac{\rho^{2n}}{\kappa^{2n}} + 1} = -\frac{2\,\rho^n \kappa^n}{\rho^{2n} + \kappa^{2n}} \sin n\theta$$

At the boundary ($\rho \to \infty$), we have

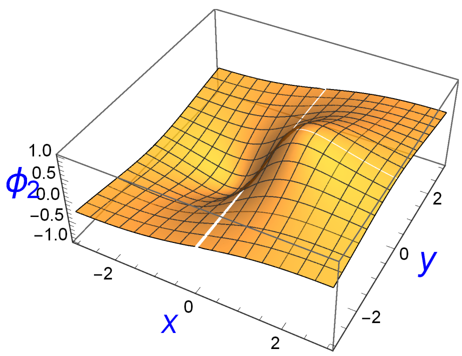
$$\boldsymbol{\phi}^\infty = (0, 0, 1)$$

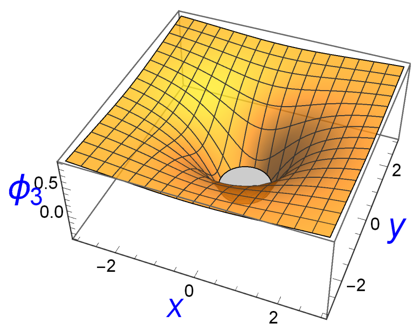
At the origin, $\rho = 0$ & we have

$$\boldsymbol{\phi}(\mathbf{0}) = (0, 0, -1)$$

Thus, we have a skyrmion sitting at the origin with a size on the order of κ.

Skyrmion with $n = 1$ & $\kappa = 1$:





Anti-Skyrmion

The anti-skyrmion version of the above is

$$\omega(z) = \left(\frac{z^*}{\kappa}\right)^n = \left(\frac{\rho}{\kappa}\right)^n e^{-i n \theta} \qquad \text{where} \qquad z = \rho\, e^{i\theta}$$

$$\omega(z)^* = \left(\frac{z}{\kappa}\right)^n = \left(\frac{\rho}{\kappa}\right)^n e^{i n \theta}$$

$$\to \qquad \phi_3 = \frac{|\,\omega\,|^2 - 1}{|\,\omega\,|^2 + 1} = \frac{\rho^{2n} - \kappa^{2n}}{\rho^{2n} + \kappa^{2n}}$$

$$\phi_1 = \frac{\omega + \omega^*}{|\,\omega\,|^2 + 1} = \frac{2\left(\frac{\rho}{\kappa}\right)^n \cos n\,\theta}{\frac{\rho^{2n}}{\kappa^{2n}} + 1} = \frac{2\,\rho^n\,\kappa^n}{\rho^{2n} + \kappa^{2n}} \cos n\,\theta$$

$$\phi_2 = i\,\frac{\omega - \omega^*}{|\,\omega\,|^2 + 1} = \frac{2\left(\frac{\rho}{\kappa}\right)^n \sin n\,\theta}{\frac{\rho^{2n}}{\kappa^{2n}} + 1} = \frac{2\,\rho^n\,\kappa^n}{\rho^{2n} + \kappa^{2n}} \sin n\,\theta$$

At the boundary ($\rho \to \infty$), we have

$$\boldsymbol{\phi}^\infty = (0, 0, 1)$$

which is the same as the skyrmion case.

At the origin, $\rho = 0$ & we have

$$\boldsymbol{\phi}(\mathbf{0}) = (0, 0, -1)$$

Again, the same as the skyrmion case.

Thus, we have an anti-skyrmion sitting at the origin with a size on the order of κ.

Anti-skyrmion with $n = 1$ & $\kappa = 1$, or skyrmion with $n = -1$ & $\kappa = 1$: