7.7.f. Physical Models

As discussed in 7.6.d._ScalingTheorem.pdf, only systems with $N \le 3$ independent real fields & a proper 2-D boundary condition can support skyrmions.

We've already demonstrated that the $O(3)$ sigma field

$$\mathcal{L} = \frac{1}{2} f\, \partial_\mu \boldsymbol{\phi} \cdot \partial^\mu \boldsymbol{\phi} \qquad\qquad \boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3)$$

support stable skyrmions in any plane (any direction can be chosen as the z-axis). It'll be used as a model for a spin QH ferromagnet.

Since the deciding factor is the boundary condition, the symmetry of the system doesn't affect the existence of skyrmions.

For example, one can down grade the $O(3)$ symmetry to $O(2)$ explicitly by adding a "Zeeman" term

$$\mathcal{L} = \frac{1}{2} f\, \partial_\mu \boldsymbol{\phi} \cdot \partial^\mu \boldsymbol{\phi} - \frac{1}{2} g_Z\, \phi_3$$

so that the type of skyrmions discussed previously can exist only in the xy plane. Other less symmetrical skyrmions may be allowed by the boundary condition. But they're expected to be less favorable energetically. This can be used to describe the spin field in a monolayer QH system.

Finally, one can break the $O(3)$ symmetry completely & set, e.g.,

$$\mathcal{L} = \frac{1}{2} f\, \partial_\mu \boldsymbol{\phi} \cdot \partial^\mu \boldsymbol{\phi} - \frac{\varepsilon_{\text{cap}}}{4} (\phi_2)^2 - \frac{1}{2} \Delta_{\text{SAS}}\, \phi_3$$

which is a model for the pseudospin fields in a spin-frozen bilayer QH system. Here, ε_{cap} & Δ_{SAS} are the capacitance & tunneling coupling constants, respectively.