4.D.0. Introduction

A quantity, whose value is given by the outcome of a random experiment, is called a stochastic (or random) variable. A stochastic variable X is therefore a real function on the sample space S, i.e.,

$$X : S \to \mathbb{R}$$

Each value that can be assumed by X is called a realization of X.

Owing to its statistical nature, a full description of X involves

(a) Identification of each event in S that gives rise to each realization.
This is achieved by defining the function X using events as its argument.
Since each measurement must result in a realization of X, these events must partition S.

(b) Assignment of a probability to each event in (a), thus turning S into a probability space.
This can be achieved by defining any one of the three following functions:

1. the probability density function.
2. the probability distribution function.
3. the characteristic function.

Note that although the concept of sample points are not necessary in the foregoing discussions, they will be, if more than one stochastic variables are defined on S. This is because realizations of different stochastic variables are invoked by different sets of events that partition S differently. Transformation between these different partitions necessarily involves the sample points.