

5.C. Markov Chains

A **Markov chain** is a Markov process at discrete times.

Consider the case where Y can take on M values $\{y(n), n = 1, \dots, M\}$, and transitions occur at times $t = s\tau$, with $s = 1, \dots, \infty$.

Let

$P(n_1, s_1)$ = Probability that $Y = y(n_1)$ at $t = s_1\tau$.

$P_{1|1}(n_1, s_1 | n_2, s_2)$ = Conditional probability that

$Y = y(n_2)$ at $t = s_2\tau$ given $Y = y(n_1)$ at $t = s_1\tau$.

Evolution of the Markov chain is completely determined by P and $P_{1|1}$.

(5.12) becomes

$$P(n, s+1) = \sum_{m=1}^M P(m, s) P_{1|1}(m, s | n, s+1) \quad (5.20)$$

The Chapman-Kolmogorov equation (5.19) takes the form

$$P_{1|1}(n_0, s_0 | n, s+1) = \sum_{m=1}^M P_{1|1}(n_0, s_0 | m, s) P_{1|1}(m, s | n, s+1) \quad (5.21)$$

Conditional probability $P_{1|1}(m, s | n, s+1)$ is called the **transition probability**. It controls how the system evolves from one time step to the next, and hence the whole process.

We define the **transition matrix** $\mathbf{Q}(s)$ as a matrix with elements

$$Q_{mn}(s) \equiv P_{1|1}(m, s | n, s+1) \quad (5.22)$$

In this section, we assume \mathbf{Q} to be time independent, i.e., $\mathbf{Q}(s) = \mathbf{Q}$.

The periodic case, $\mathbf{Q}(s+N) = \mathbf{Q}(s)$, will be discussed in §S5.A.