7.H.0. Introduction

For a system of N non-interacting particles, the Hamiltonian is

$$\hat{H} = \sum_{i=1}^{N} \hat{h}_i$$

where the 1-particle Hamiltonian $\hat{h}$ satisfies the Schrodinger equation

$$\hat{h} \mid l \rangle = \epsilon_l \mid l \rangle \qquad (7.119)$$

where l represents all the quantum numbers necessary to describe the 1-particle eigenstate.

Due to the uncertainty principle, identical particles in close vicinity of each other become indistinguishable. Their physical properties are therefore invariant under particle exchanges. The total wave functions must then be either symmetric or anti-symmetric under particle exchanges, which corresponds to bosons or fermions, respectively.

In the absence of particle-particle interactions, eigenstates of the system are simply symmetric or antisymmetric combinations of the products of N single particle eigenstates. Let there be n_l particles in the 1-particle eigenstate $\mid l \rangle$ of energy ϵ_l. The total energy of the system is simply

$$E\left(\{n_l\} \right) = \sum_l n_l \, \epsilon_l \qquad (7.120)$$

for both bosons & fermions. Although there are no restrictions on n_l for bosons, the antisymmetry of fermions limits n_l to either 0 or 1.

Using the properly symmetrized eigenstates as basis, the grand partition function

$$Z_\mu(T, V) = \text{Tr}\left[e^{-\beta\left(\hat{H} - \mu' \hat{N}\right)} \right] \qquad (7.118)$$

simplifies to

$$Z_{\text{BE}}(T, V, \mu') = \prod_l \left(\sum_{n_l=0}^{\infty} e^{-\beta n_l (\epsilon_l - \mu')} \right) \qquad \text{for bosons.} \qquad (7.121)$$

$$Z_{\text{FD}}(T, V, \mu') = \prod_l \left(\sum_{n_l=0}^{1} e^{-\beta n_l (\epsilon_l - \mu')} \right) \qquad \text{for fermions.} \qquad (7.123)$$