10.C.2. Transverse Hydrodynamic Modes

Given $\boldsymbol{v}_{\boldsymbol{k}}^{\perp}(0)$, (10.66) can be solved to give

$$\tilde{\boldsymbol{v}}_{\boldsymbol{k}}^{\perp}(z) = \frac{\boldsymbol{v}_{\boldsymbol{k}}^{\perp}(0)}{z + \nu_t k^2} \tag{10.67}$$

The Laplace inverse transform (10.55) becomes

$$\begin{aligned}\tilde{\boldsymbol{v}}_{\boldsymbol{k}}^{\perp}(t) &= \frac{1}{2\pi i} \int_{-i\infty+\delta}^{i\infty+\delta} dz\, e^{zt} \frac{\boldsymbol{v}_{\boldsymbol{k}}^{\perp}(0)}{z + \nu_t k^2} \\ &= e^{-\nu_t k^2 t}\, \boldsymbol{v}_{\boldsymbol{k}}^{\perp}(0)\end{aligned} \tag{10.68}$$

where $\delta = 0$ since the only pole is at $z = -\nu_t k^2$.

Thus, $\tilde{\boldsymbol{v}}_{\boldsymbol{k}}^{\perp}$ decays with relaxation time $\tau_t = \dfrac{1}{\nu_t k^2}$ proportional to λ^2. This wavelength dependence is the signature of the transverse hydrodynamic mode.