Read Reichl's §11.A.

11.B.1. The Maxwell-Boltzmann Distribution

Consider an ideal gas of N-particles in a volume V with Hamiltonian

$$H = \sum_{i=1}^{N} \frac{\boldsymbol{p}_i^2}{2m} \tag{11.1a}$$

In a canonical ensemble, the probability density is

$$\rho(\boldsymbol{q}^N, \boldsymbol{p}^N) = \frac{1}{Z} \exp\left(-\beta \sum_{i=1}^{N} \frac{\boldsymbol{p}_i^2}{2m}\right) \tag{11.1}$$

where

$$Z = \int d\,\boldsymbol{q}^N \int d\,\boldsymbol{p}^N \exp\left(-\beta \sum_{i=1}^{N} \frac{\boldsymbol{p}_i^2}{2m}\right) \tag{11.1b}$$

is the partition function and

$$d\,\boldsymbol{q}^N\, d\,\boldsymbol{p}^N = d\,\boldsymbol{q}_1 \ldots d\,\boldsymbol{q}_N\, d\,\boldsymbol{p}_1 \ldots d\,\boldsymbol{p}_N = \prod_{i=1}^{N} d\,\boldsymbol{q}_i\, d\,\boldsymbol{p}_i \tag{11.1c}$$

$\rho(\boldsymbol{q}^N, \boldsymbol{p}^N)\, d\,\boldsymbol{q}^N\, d\,\boldsymbol{p}^N$ = probability of finding system inside volume element $d\,\boldsymbol{q}^N\, d\,\boldsymbol{p}^N$ centered at point $(\boldsymbol{q}^N, \boldsymbol{p}^N)$ in phase space.

The 1-particle momentum probability density is defined as

$$\rho_1(\boldsymbol{p}_1) = \int d\,\boldsymbol{q}^N \int d\,\boldsymbol{p}^{N-1}\, \rho(\boldsymbol{q}^N, \boldsymbol{p}^N)$$

$$= \frac{\exp\left(-\beta \frac{\boldsymbol{p}_1^2}{2m}\right)}{\int d\,\boldsymbol{p}_1 \exp\left(-\beta \frac{\boldsymbol{p}_1^2}{2m}\right)} = \left(\frac{\beta}{2\pi m}\right)^{3/2} \exp\left(-\beta \frac{\boldsymbol{p}_1^2}{2m}\right) \tag{11.2}$$

with

$\rho_1(\boldsymbol{p}_1)\, d\,\boldsymbol{p}_1$ = probability of finding a particle with momentum inside $d\,\boldsymbol{p}_1$ centered at $\boldsymbol{p}_1$.

Setting $\boldsymbol{p} = m\,\boldsymbol{v}$, we find the 1-particle velocity probability density to be

$$F(\boldsymbol{v}_1) = \frac{\exp\left(-\beta \frac{1}{2} m\, \boldsymbol{v}_1^2\right)}{\int d\,\boldsymbol{v}_1 \exp\left(-\beta \frac{1}{2} m\, \boldsymbol{v}_1^2\right)} = \left(\frac{m\beta}{2\pi}\right)^{3/2} \exp\left(-\beta \frac{1}{2} m\, \boldsymbol{v}_1^2\right) \tag{11.3}$$

which is known as the Maxwell-Boltzmann distribution.

Obviously,

$$\int d\,\boldsymbol{p}_1\, \rho_1(\boldsymbol{p}_1) = \int d\,\boldsymbol{v}_1\, F(\boldsymbol{v}_1) = 1 \tag{11.3a}$$