11.B.2. The Mean Free Path

The mean free path λ is the average distance a particle can travel between collisions.

Assuming collisions occur at random in a gas, the probability of a collision is the same during any given distance $d\,r$ traveled, wherever it happens. Therefore,

$$\frac{d\,r}{\lambda} = \text{probability that a collision occurs after a distance } d\,r \text{ is traveled.}$$

$$1 - \frac{d\,r}{\lambda} = \text{probability that no collision occurs after a distance } d\,r \text{ is traveled.}$$

Let

$$P_0(r) = \text{probability that no collision occurs after a distance } r \text{ is traveled.}$$

then

$$P_0(r + d\,r) = P_0(r)\left(1 - \frac{d\,r}{\lambda}\right) \tag{11.4}$$

$$\rightarrow \quad \frac{d\,P_0(r)}{d\,r} = \lim_{d\,r \to 0} \frac{P_0\,(r + d\,r) - P_0(r)}{d\,r} = -\frac{1}{\lambda}\,P_0(r) \tag{11.5}$$

By definition

$$P_0(0) = 1 \tag{11.6a}$$

The solution to (11.5) satisfying (11.6a) is

$$P_0(r) = e^{-r/\lambda} \tag{11.6}$$

which is simply the Poisson distribution for zero outcome [see (4.60) of §4.E.3 with $n_1 = 0$].

Hence,

$$P_0(r)\,\frac{d\,r}{\lambda} = \text{probability that the 1st collision occurs in interval } (\,r,\ r + d\,r\,)$$

The average distance traveled before a collision occurs is therefore

$$\langle\, r \,\rangle = \int_0^\infty r\,P_0(r)\,\frac{d\,r}{\lambda} \tag{11.7}$$

$$= \int_0^\infty r\,e^{-r/\lambda}\,\frac{d\,r}{\lambda} \qquad \text{[(11.6) used.]}$$

$$= \lambda \qquad \text{[See } \textit{Mathematica} \text{ code below.]}$$

in agreement with the definition of λ.

------------------------ *Mathematica* code --

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Assuming[λ > 0, ∫_0^∞ r ⅇ^(-r/λ) ⅆr]
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λ^2
