11.B.5. The Coefficients of Viscosity and Thermal Conductivity

Any macroscopic property A that is out of equilibrium will relax toward equilibrium. The driving force here is therefore $-\nabla_{\boldsymbol{r}} A$.

In order to concentrate on the effects due purely to $\nabla_{\boldsymbol{r}} A$, we assume the average particle density n of the system to be a constant [c.f. the case of self-diffusion].

Assuming a particle at $\boldsymbol{r}$ carries an amount $A(\boldsymbol{r})$ of the property A, we use the same arguments that led to (11.18) to get

$$d\dot{A}_+ = A(\boldsymbol{r})\left(\frac{\langle v\rangle}{\lambda} n\, d V_+\right)\left(\frac{|\cos\theta|}{4\pi r^2} d S\right) e^{-r/\lambda}$$

= change of A at $d S$ due to particles hitting $d S$ from $d V_+$ per unit time.

This means the results of §11.B.4 can be adapted to the present discussion by the substitution

$$n_T(\boldsymbol{r}) \;\to\; n\, A(\boldsymbol{r})$$

In particular, (11.25) becomes

$$\boldsymbol{J}_A(\boldsymbol{r}) = -\frac{\langle v\rangle\lambda}{3} n \nabla_{\boldsymbol{r}} A(\boldsymbol{r}) \qquad (10.30)$$

= Flux of A at $\boldsymbol{r}$.

11.B.5.1. Self-Diffusion

Obviously, setting

$$A(\boldsymbol{r}) = \frac{n_T(\boldsymbol{r})}{n}$$

gives us the self-diffusion results.

11.B.5.2. Viscosity

Viscosity is the measure of friction between adjacent fluid elements that move with different velocities. In particular, the symmetric part of the stress tensor is related to the coefficient of viscosity η by [see (10.30) of §10.B.3],

$$\boldsymbol{\Pi}^s = -2\,\eta\,[\nabla_{\boldsymbol{r}} \boldsymbol{v}]^s$$

Setting

$$A_i(\boldsymbol{r}) = m\, v_i(\boldsymbol{r}) \quad \text{or} \qquad \boldsymbol{A}(\boldsymbol{r}) = m\, \boldsymbol{v}(\boldsymbol{r})$$

(10.30) becomes

$$\boldsymbol{J}_{A_i}(\boldsymbol{r}) = -\frac{\langle v\rangle\lambda}{3} n\, m \nabla_{\boldsymbol{r}} v_i(\boldsymbol{r})$$

or

$$J_{j A_i}(\boldsymbol{r}) \equiv J_{j i}(\boldsymbol{r}) = -\frac{\langle v\rangle\lambda}{3} n\, m\, \partial_j v_i(\boldsymbol{r})$$

$$\to \qquad \mathbb{J} = -\frac{\langle v\rangle\lambda}{3} n\, m \nabla_r \boldsymbol{v} = -\eta \nabla_r \boldsymbol{v} \qquad (11.32)$$

where

$$\eta = \frac{\langle v \rangle \lambda}{3} n m \qquad (11.33)$$

Using (11.17), we have

$$\eta = \frac{m \langle v \rangle}{3 \sqrt{2} \pi d^2} \quad \text{for hard spheres of diameter } d \qquad (11.34)$$

11.B.5.3. Heat Conduction

For an ideal gas, heat is just the average kinetic energy. Therefore,

$$A(\boldsymbol{r}) = \frac{1}{2} m \langle v^2(\boldsymbol{r}) \rangle = \frac{3}{2} k_B T(\boldsymbol{r}) \text{ [Equipartition theorem used.]} \qquad (11.35)$$

and (10.30) becomes

$$\boldsymbol{J}_Q(\boldsymbol{r}) \equiv \boldsymbol{J}_A(\boldsymbol{r}) = -\frac{\langle v \rangle \lambda}{2} n k_B \nabla_{\boldsymbol{r}} T(\boldsymbol{r}) = -K \nabla_{\boldsymbol{r}} T(\boldsymbol{r}) \qquad (11.36)$$

where

$\boldsymbol{J}_Q$ = heat flux

$$K = \frac{\langle v \rangle \lambda}{2} n k_B = \text{coefficient of thermal conductivity} \qquad (11.37)$$

Using

$$\tilde{c}_V = \frac{1}{m} \left(\frac{\partial Q}{\partial T} \right)_V = \text{specific heat}$$

$$= \frac{3}{2m} k_B \qquad \text{[(11.35) used.]}$$

(11.37) becomes

$$K = \frac{\langle v \rangle \lambda}{3} n m \tilde{c}_V \qquad (11.38)$$

Comparing (10.38) with (11.33) gives

$$K \approx \eta \tilde{c}_V \qquad \text{for ideal gas} \qquad (11.39)$$