11.F.3. Shear Viscosity and Thermal Conductivity

Matching the eigen-frequencies in (11.181-5) to those of the linearized hydrodynamic equations (11.151-4), we obtain microscopic expressions for the transport coefficients.

For example, with $\hat{\boldsymbol{k}} = \hat{\boldsymbol{x}}$, equating (11.151) with (11.184) gives,

$$\nu_t = \frac{\eta}{m\, n_0} = -\left\langle \Phi_{05}^{(0)} \;\middle|\; \frac{p_x}{m} \frac{1}{\hat{C}^+} \frac{p_x}{m} \;\middle|\; \Phi_{05}^{(0)} \right\rangle \qquad \text{[(11.150a) used.]}$$

(11.186a)

Using (11.178) & (11.160), we can write (11.186a) as

$$\eta = -m\, n_0 \left(\frac{\beta}{2\pi m} \right)^{3/2} \int d\boldsymbol{p}\; e^{-\beta p^2/2m} \left(\sqrt{\frac{\beta}{m}}\, p_y \right) \frac{p_x}{m} \frac{1}{\hat{C}^+} \frac{p_x}{m} \left(\sqrt{\frac{\beta}{m}}\, p_y \right)$$

$$= -\frac{n_0\, \beta}{m^2} \left(\frac{\beta}{2\pi m} \right)^{3/2} \int d\boldsymbol{p}\; e^{-\beta p^2/2m}\, p_y\, p_x \frac{1}{\hat{C}^+} p_x\, p_y \qquad (11.186)$$

Similarly, equating (11.153) with (11.185) gives

$$\chi = \frac{K}{n_0\, c_P} = -\left\langle \Phi_{03}^{(0)} \;\middle|\; \frac{p_x}{m} \frac{1}{\hat{C}^+} \frac{p_x}{m} \;\middle|\; \Phi_{03}^{(0)} \right\rangle$$

(11.187a)

Combining (11.180), (11.158) & (11.162), we have

$$\Phi_{03}^{(0)}(\boldsymbol{p}) = -\sqrt{\frac{2}{5}} + \sqrt{\frac{3}{5}} \sqrt{\frac{2}{3}} \left(-\frac{3}{2} + \frac{\beta}{2m} p^2 \right)$$

$$= \sqrt{\frac{2}{5}} \left(-\frac{5}{2} + \frac{\beta}{2m} p^2 \right)$$

so that (11.187a) becomes

$$K = -n_0\, c_P \left(\frac{\beta}{2\pi m} \right)^{3/2} \int d\boldsymbol{p}\; e^{-\beta p^2/2m} \sqrt{\frac{2}{5}} \left(-\frac{5}{2} + \frac{\beta}{2m} p^2 \right) \frac{p_x}{m} \frac{1}{\hat{C}^+} \frac{p_x}{m} \sqrt{\frac{2}{5}} \left(-\frac{5}{2} + \frac{\beta}{2m} p^2 \right)$$

$$= -\frac{2\, n_0\, c_P}{5\, m^2} \left(\frac{\beta}{2\pi m} \right)^{3/2} \int d\boldsymbol{p}\; e^{-\beta p^2/2m} \left(-\frac{5}{2} + \frac{\beta}{2m} p^2 \right) p_x \frac{1}{\hat{C}^+} p_x \left(-\frac{5}{2} + \frac{\beta}{2m} p^2 \right) \qquad (11.187)$$

Setting

$$J_{xy} = \frac{1}{m} p_x\, p_y = p_x\, v_y = \text{flux of } p_x \text{ in the } y\text{-direction} \qquad (11.187a)$$

we can write (11.186) as

$$\eta = -n_0\, \beta \left(\frac{\beta}{2\pi m} \right)^{3/2} \int d\boldsymbol{p}\; e^{-\beta p^2/2m}\, J_{xy} \frac{1}{\hat{C}^+} J_{xy}$$

(11.187b)

Using

$$\int_0^\infty d\tau\, e^{\hat{C}^+ \tau} = -\frac{1}{\hat{C}^+}$$

(11.197b) becomes

$$\eta = n_0 \beta \left(\frac{\beta}{2\pi m}\right)^{3/2} \int_0^\infty d\tau \int d\boldsymbol{p}\, e^{-\beta p^2/2m}\, J_{xy}\, e^{\hat{C}^+ \tau} J_{xy} \tag{11.188}$$

$$= n_0 \beta \left(\frac{\beta}{2\pi m}\right)^{3/2} \int_0^\infty d\tau \int d\boldsymbol{p}\, e^{-\beta p^2/2m}\, J_{xy}\, J_{xy}(\tau)$$

where

$$J_{xy}(\tau) = e^{\hat{C}^+ \tau} J_{xy}$$

(11.189)

Defining the momentum flux correlation function as

$$\langle J_{xy}\, J_{xy}(\tau) \rangle_{eq} = \left(\frac{\beta}{2\pi m}\right)^{3/2} \int d\boldsymbol{p}\, e^{-\beta p^2/2m}\, J_{xy}\, J_{xy}(\tau) \tag{11.190a}$$

$$= \langle J_{xy},\ J_{xy}(\tau) \rangle$$

(11.188) becomes

$$\eta = n_0 \beta \int_0^\infty d\tau \, \langle J_{xy}\, J_{xy}(\tau) \rangle_{eq} \tag{11.190}$$

Similarly, K can be expressed in terms of the enthalpy flux correlation function.

Note that this relation between transport coefficients and correlation functions is just another expression of the dissipation-fluctuation theorem discussed in Chap 10.