12.B. Non-equilibrium Stability Criteria

Read §12.A in Reichl's text.

Consider a closed isolated system of fixed volume of surface $\boldsymbol{S}$. Using [see (10.23)]

$$\boldsymbol{J}_s^R = \rho\, s\, \boldsymbol{v}$$

the entropy balance equation (10.10) becomes

$$\frac{\partial \rho s}{\partial t} + \nabla_r \cdot \left(\rho\, s\, \boldsymbol{v} + \boldsymbol{J}_s^D \right) = \sigma_s \tag{12.1a}$$

which can be integrated over V to give

$$\begin{aligned} \frac{dS}{dt} &= \int_V d\boldsymbol{r} \frac{\partial \rho s}{\partial t} \\ &= -\int_V d^3 r\, \nabla_r \cdot \left(\rho\, s\, \boldsymbol{v} + \boldsymbol{J}_s^D \right) + \int_V d^3 r\, \sigma_s \\ &= -\int_{\boldsymbol{S}} d\,\boldsymbol{S} \cdot \left(\rho\, s\, \boldsymbol{v} + \boldsymbol{J}_s^D \right) + \int_V d^3 r\, \sigma_s \end{aligned} \tag{12.1}$$

Since the system is closed and isolated, no matter or energy (and hence entropy) can flow through the surface. (12.1) thus reduces to

$$\frac{dS}{dt} = \int_V d^3 r\, \sigma_s \tag{12.2}$$

= rate of total entropy production due to dissipative processes inside V

On the other hand, the 2nd law states that, for an isolated system,

$$\frac{dS}{dt} \geq 0 \tag{12.3}$$

Since (12.3) holds for arbitrary V, we have

$$\sigma_s \geq 0 \tag{12.4}$$

As in (10.26) & (10.179), the local entropy production takes the form [see (10.26) or (10.81)]

$$T\, \sigma_s = -\sum_i \mathcal{J}_i\, \chi_i \tag{12.4a}$$

where $\mathcal{J}_i$ is a local flux and χ_i a local force.