S3.B. Thermomechanical Effect

Below $T = 2.19\,K$, which is the highest temperature on the λ-line [see Fig.3.15 of §3.F.1], liquid $^4\mathrm{He}$ is composed of a normal (He *I*) and a superfluid (He II) component. Since He II is a Bose condensate, its flow is frictionless and carries no entropy; thus leading to some unusual behavior. For example, He II can flow through cracks too small for He *I* to leak through.

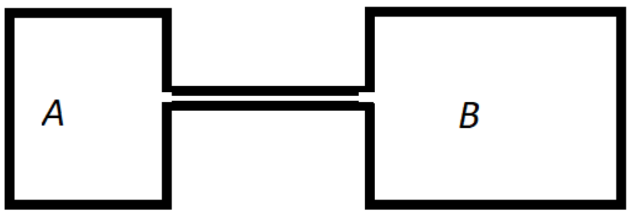

Fig.3.29.

Fig.3.29 shows two insulated rigid vessels A & B filled, without leak, with liquid $^4\mathrm{He}$ at $T < 2.19\,K$ and connected by an insulated capillary narrow enough so that only He II can pass. Thus, the total mass of the system remain constant. So is the total entropy if no irreversible processes occur.

Using the subscript $\alpha = A, B$ to denote quantities in vessel α, the total internal energy can be written as

$$U_T = \sum_{\alpha = A, B} M_\alpha \tilde{u}_\alpha \tag{3.101}$$

where M and $\tilde{u}$ are the mass and specific internal energy (internal energy per unit mass), respectively. Since the system is isolated (no mass, heat, nor work exchange with environment),

$$\delta U_T = 0$$

$$= \sum_{\alpha = A, B} (M_\alpha \delta \tilde{u}_\alpha + \tilde{u}_\alpha \delta M_\alpha) \qquad \text{[(3.101) used.]} \tag{3.102}$$

Now, He *I*, and hence entropy, cannot pass between the vessels. Therefore,

$$S_\alpha = M_\alpha \tilde{s}_\alpha = \text{constant}$$

where $\tilde{s}$ is the specific entropy (entropy per unit mass). Hence,

$$\delta S_\alpha = 0 = M_\alpha \delta \tilde{s}_\alpha + \tilde{s}_\alpha \delta M_\alpha \tag{3.102a}$$

Similarly, since the vessels are rigid,

$$\delta V_\alpha = 0 = M_\alpha \delta \tilde{v}_\alpha + \tilde{v}_\alpha \delta M_\alpha \tag{3.102b}$$

where $\tilde{v}$ is the specific volume (volume per unit mass).

(3.102a) and (3.102b) give

$$\delta \tilde{s}_\alpha = -\tilde{s}_\alpha \frac{\delta M_\alpha}{M_\alpha} \tag{3.103}$$

$$\delta \tilde{v}_\alpha = -\tilde{v}_\alpha \frac{\delta M_\alpha}{M_\alpha} \tag{3.104}$$

Taking

$$\tilde{u} = \tilde{u}\left(\tilde{s}, \tilde{v}\right)$$

we have

$$\delta \tilde{u} = \left(\frac{\partial \tilde{u}}{\partial \tilde{s}}\right)_{\tilde{v}} \delta \tilde{s} + \left(\frac{\partial \tilde{u}}{\partial \tilde{v}}\right)_{\tilde{s}} \delta \tilde{v}$$

so that (3.102) becomes

$$\sum_{\alpha=A,B} \left\{ M_\alpha \left[\left(\frac{\partial \tilde{u}_\alpha}{\partial \tilde{s}_\alpha}\right)_{\tilde{v}_\alpha} \delta \tilde{s}_\alpha + \left(\frac{\partial \tilde{u}_\alpha}{\partial \tilde{v}_\alpha}\right)_{\tilde{s}_\alpha} \delta \tilde{v}_\alpha \right] + \tilde{u}_\alpha \delta M_\alpha \right\} = 0 \qquad (3.105)$$

$$\rightarrow \quad \sum_{\alpha=A,B} \left[M_\alpha \left(T_\alpha \delta \tilde{s}_\alpha - P_\alpha \delta \tilde{v}_\alpha \right) + \tilde{u}_\alpha \delta M_\alpha \right] = 0 \qquad \text{[(2.68-9) used.]}$$

$$\sum_{\alpha=A,B} \left[M_\alpha \left(-T_\alpha \tilde{s}_\alpha \frac{\delta M_\alpha}{M_\alpha} + P_\alpha \tilde{v}_\alpha \frac{\delta M_\alpha}{M_\alpha} \right) + \tilde{u}_\alpha \delta M_\alpha \right] = 0 \qquad \text{[(3.103-4) used.]}$$

$$\therefore \quad \sum_{\alpha=A,B} \left(-T_\alpha \tilde{s}_\alpha + P_\alpha \tilde{v}_\alpha + \tilde{u}_\alpha \right) \delta M_\alpha = 0 \qquad (3.106)$$

The fundamental equation (2.61) of § 2.E

$$U = T S - P V + \mu' N$$

can be written as

$$\tilde{u} = T \tilde{s} - P \tilde{v} + \tilde{\mu} \qquad (3.106a)$$

where

$\tilde{\mu} = \dfrac{\mu' N}{M} = \dfrac{\mu'}{m}$ = specific chemical potential (chemical potential per unit mass)

$m = \dfrac{M}{N}$ = mass of one particle

Putting (3.106a) into (3.106) gives

$$\sum_{\alpha=A,B} \tilde{\mu}_\alpha \delta M_\alpha = 0 \qquad (3.106b)$$

Since there is no leakage to the environment,

$$M_T = \sum_{\alpha=A,B} M_\alpha = \text{constant}$$

$$\rightarrow \quad \delta M_A + \delta M_B = 0$$

so that (3.106b) implies

$$\tilde{\mu}_A (T_A, P_A) = \tilde{\mu}_B (T_B, P_B) \qquad (3.107)$$

which is just the equilibrium condition between systems that can exchange particles.

Since no heat nor work can be exchanged between the vessels, equilibrium does not require T & P in them to be equal.

Consider now a change in T_A & P_A. The corresponding change in $\tilde{\mu}_A$ is given by the Gibbs-Duhem equation as

$$\Delta \tilde{\mu}_A = -\tilde{s}_A \Delta T_A + \tilde{v}_A \Delta P_A \qquad (3.108)$$

By (3.107), equilibrium with vessel B will not be affected if

$$\Delta \tilde{\mu}_A = 0$$

namely,

$$\Delta P_A = \frac{\tilde{s}_A}{\tilde{v}_A} \Delta T_A \qquad (3.109)$$

Thus, a rise in T_A will be accompanied by a rise in P_A to keep vessel B undisturbed. This is called the thermomechanical effect and is the cause of the fountain effect shown in Fig.3.30a.

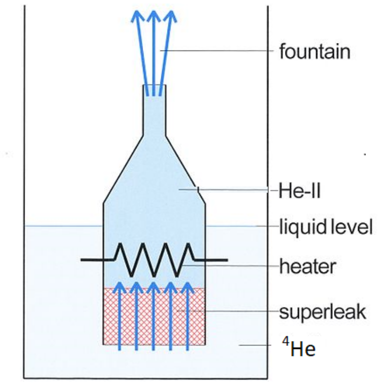

Fig.3.30a. Fountain effect. Only He II can pass through the super-leak plug. When He II is heated, it erupts out of the opening at the top due to the rise in pressure prescribed by (3.109).