S3.D. Tri-critical Points

A tricritical point is where the critical point of a discontinuous transition meets the λ-line of continuous transitions [see Fig.3.19 of §3.F.3].

Comparing with Fig.3.17, we see that in the temperature scale of Fig.3.19, the ^{3}He component can be taken as consisting of only normal liquid. Hence, liquid ^{3}He-^{4}He is a mixture of 3 components, ^{3}He , He *I*, & He II. Let η be the order parameter for the He *I*-He II continuous transition. The molar Ginzburg-Landau free energy then takes the form [see (k4) of §3.G]

$$\phi(T, P, \eta, x_3) = x_4 \alpha_4 \left[\tilde{\phi}_0(T, P) + \frac{3}{16} \Delta \eta^2 + \frac{1}{4!} \eta^4 \right] + x_3 \mu_3^0(T, P) + R T \left(x_4 \ln x_4 + x_3 \ln x_3 \right) + \lambda x_3 x_4$$

(3.114a)

$$= x_4 \mu_4(T, P, \eta) + x_3 \mu_3^0(T, P) + R T \left(x_4 \ln x_4 + x_3 \ln x_3 \right) + \lambda x_3 x_4$$

where x_3 & x_4 are the concentration of ^{3}He & ^{4}He , respectively, with

$$x_3 + x_4 = 1$$

λ is a parameter such that, if μ_4 is independent of x_4, the critical temperature for the binary mixture is [see Ex.3.6 of §S3.E]

$$T_c = \frac{\lambda}{2R}$$

(3.114b)

As usual, the equilibrium phase is given by the global minimum of ϕ with respect to both η & x_3.

For the sake of clarity, let us ignore the interactions between the two transitions. (3.114b) then holds while the transition points of the He *I*-He II transition are given by

$$\Delta = \Delta_X = 0$$

(3.114c)

Taking $\Delta = \Delta(T, P)$, (3.114c) is the equation for the λ-line in the T-P plane. For $T > T_c$, the ^{3}He-^{4}He mixture is in a single phase so that the transition is purely continuous. However, for $T < T_c$, the ^{3}He-^{4}He mixture is a coexistence of the pure ^{3}He and pure ^{4}He phases. The λ-line , as viewed in the x_3-T plane, is then spread out into a coexistence region [see Fig.3.19]. The transition between normal ^{4}He & superfluid ^{4}He then becomes discontinuous even though the He *I*-He II transition remains continuous.

The tri-critical point is simply the critical point of the binary mixture.

Bringing back the interactions between the two transitions should only modify the numerical results but not the qualitative features of the answer.