Chapter 10
Due: 12:00am on Saturday, July 3, 2010
Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy.

Kinetic Energy of a Rotating Wheel

Description: A simple application of the formula for rotational kinetic energy of a rigid body. Compute both moment of inertia and angular velocity from given information.

A simple wheel has the form of a solid cylinder of radius \( r \) with a mass \( m \) uniformly distributed throughout its volume. The wheel is pivoted on a stationary axle through the axis of the cylinder and rotates about the axle at a constant angular speed. The wheel rotates \( n \) full revolutions in a time interval \( t \).

Part A
What is the kinetic energy \( K \) of the rotating wheel?

**Hint A.1** What is the formula for rotational kinetic energy?
Express the rotational kinetic energy in terms of \( I \), the moment of inertia of the wheel, and \( \omega \), its angular velocity.

**ANSWER:**

\[ K = \frac{1}{2} I \omega^2 \]

**Hint A.2** Moment of inertia of the wheel
For a solid cylinder of mass \( m \), radius \( r \), and any height rotating about its axis, the moment of inertia is given by

\[ I = \frac{1}{2} mr^2. \]

This formula is the same as that for the moment of inertia for a solid disk of mass \( m \) and radius \( r \) rotating about its center of mass. (Why?)

**Hint A.3** Find the angular velocity
What is the angular velocity \( \omega \) of the wheel in radians per second?

**Hint A.3.1** Relationship between revolutions and radians
Recall that 1 revolution = \( 2\pi \) radians.

Express your answer in terms of quantities given in the problem introduction and \( \pi \).

**ANSWER:**

\[ \omega = \frac{2\pi n}{t} \text{ radians/second} \]

Express your answer in terms of \( m \), \( r \), \( n \), \( t \) and, \( \pi \).

**ANSWER:**

\[ K = \frac{mr^2n^2\pi^2}{t^2} \]

Rotational Kinetic Energy and Conservation of Energy Ranking Task

Description: Conceptual question involving conservation of energy and rotational kinetic energy. (ranking task)
The five objects of various masses, each denoted \( \text{TE} \), all have the same radius. They are all rolling at the same speed as
they approach a curved incline.

**Part A**
Rank the objects based on the maximum height they reach along the curved incline.

**Hint A.1 Using energy conservation**
As the objects roll up the incline, their initial kinetic and rotational kinetic energy gets converted into gravitational potential energy. Both kinetic and rotational kinetic energy are directly proportional to mass. Thus, objects with large mass will have proportionally more energy than objects with low mass. The same is true for gravitational potential energy.

Notice that when you equate the initial energy and the final potential energy, you are equating two quantities that are both proportional to the mass. Therefore, the mass will cancel from the equation.

**Hint A.2 Moment of inertia**
Objects with larger moments of inertia \( I \) are harder to rotate. Since all of the objects are initially rotating at the same speed (since they are traveling at the same speed and have the same radius), large-\( I \) objects will have more energy than low-\( I \) objects of equal mass.

Rank from largest to smallest. To rank items as equivalent, overlap them.

**ANSWER:**

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**Problem 10.36**
**Description:** Humanity uses energy at the rate of about 10^{13} W. (a) If we found a way to extract this energy from Earth’s rotation, how long would it take before the length of the day increased by 1 minute?

Humanity uses energy at the rate of about 10^{13} W.

**Part A**
If we found a way to extract this energy from Earth’s rotation, how long would it take before the length of the day increased by 1 minute?

Express your answer using one significant figure.

**ANSWER:**

\[ t = 4 \times 10^{13} \text{ s} \]

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**Problem 10.33**
**Description:** A 107 g frisbee is 35 cm in diameter and has about half its mass spread uniformly in a disk and the other half concentrated in the rim. With a quarter-turn flick of the wrist, a student sets the frisbee rotating at 88 rpm. (a) What is the rotational...

A 107 g frisbee is 35 cm in diameter and has about half its mass spread uniformly in a disk and the other half concentrated.
in the rim. With a quarter-turn flick of the wrist, a student sets the Frisbee rotating at 450 rpm.

**Part A**
What is the rotational inertia of the Frisbee?
Express your answer using two significant figures.

**ANSWER:**
\[ I = \frac{3}{4} M \left( \frac{d}{2} \right)^2 \text{ kg} \cdot \text{m}^2 \]

**Part B**
What is the magnitude of the torque, assumed constant, that the student applies?
Express your answer using two significant figures.

**ANSWER:**
\[ \tau = \frac{3}{4} M \left( \frac{d}{2} \right)^2 \omega^2 \frac{\pi}{\tau} \text{ N} \cdot \text{m} \]

**Problem 10.55**

**Description:** A space station is constructed in the shape of a wheel \(21\text{ in.}\) in diameter, with essentially all of its \(6.7 \times 10^5\) kg mass at the rim. Once the station is completed, it is set rotating at a rate that requires an object at the rim to have radial acceleration \(g\), thereby simulating Earth's surface gravity. This is accomplished using two small rockets, each with \(150\) N thrust, that are mounted on the rim of the station.

**Part A**
How long will it take to reach the desired spin rate?
Express your answer using two significant figures.

**ANSWER:**
\[ t = \frac{\sqrt{\frac{2gM}{d}}} {4F} \text{ s} \]

**Part B**
How many revolutions will the station make in this time?
Express your answer using two significant figures.

**ANSWER:**
\[ \Delta \theta = \frac{2\pi \frac{gM}{d}}{2} \text{ rev} \]

**Problem 10.57**

**Description:** A \(3.0\) kg block rests on a 30 degree(s) slope and is attached by a string of negligible mass to a solid drum of mass \(0.75\) kg and radius \(3.8\) cm, as shown in the figure. When released, the block accelerates down the slope at \(2.0\) m/s^2.

A 3.0 kg block rests on a 30° slope and is attached by a string of negligible mass to a solid drum of mass 0.75 kg and radius 3.8 cm, as shown in the figure. When released, the block accelerates down the slope at 2.0 m/s^2.
Part A

What is the coefficient of friction between block and slope?

Express your answer using two significant figures.

\[
\mu_k = \tan \left( \frac{\pi}{6} \right) - \frac{a \left( m + \frac{1}{2} \right)}{m \cdot 9.8 \cos \left( \frac{\pi}{6} \right)}
\]

Problem 10.75

Description: (a) Your team is helping to design an orbiting space station. The station will be in the shape of a wheel 30 m in diameter with essentially all of its 4 × 10^5 kg mass contained in the rim portion of the wheel (the mass of the spokes is negligible). Once...

Part A

Your team is helping to design an orbiting space station. The station will be in the shape of a wheel 30 m in diameter with essentially all of its 4 × 10^5 kg mass contained in the rim portion of the wheel (the mass of the spokes is negligible). Once the station is completed, it will rotate at a rate such that a person working in the wheel will experience a simulated gravitational force equal to that of Earth; that is, the normal force of the floor on an occupant would be equal to \( mg \) \( (g = 9.8 \text{ m/s}^2) \). When the station is finished, it will be necessary to set it rotating at the required rotation rate to produce this “artificial gravity.” This is accomplished by firing four small thruster rockets attached symmetrically to the outer wall of the wheel. The rockets will fire for 9 hours to bring the station “up to speed.” Your task is to determine the thrust, or force, each rocket must have to produce the required rotation rate in this amount of time. You may assume the station starts rotating from rest.

Express your answer using one significant figure.

\[
F = \frac{M}{4t} \sqrt{\frac{9.8d}{2}} \text{ N}
\]