Chapter 12
Due: 12:00am on Saturday, July 3, 2010
Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

An Eccentric Professor

Description: Determine the normal force exerted on an object in static equilibrium in an odd setup. Then, find the amount of load that must be added to break equilibrium. Uses applets.

Realizing that she often doesn't have her students' full attention during class, a professor devises an elaborate device on which to stand while she lectures. The pulley is placed so that the string makes a 45.0-degree angle with the beam. The beam is uniform, 5.00 meters long, and has weight \( W_b \). The professor stands 2.00 meters from the pivot point and has weight \( W_p \).

Part A
Find the tension \( T \) in the rope in terms of the professor's weight \( W_p \) and the weight of the beam \( W_b \).

Hint A.1 How to approach the problem
Since the professor and her contraption are in static equilibrium, the net torque on the beam must be zero. You should pick a convenient point around which to calculate the net torque.

Ideally, you should pick a point at which the force exerted on the beam is neither one that you are given nor one that you are seeking. The torque about some point due to any force exerted at that point is zero:

\[ r = d|F|\sin(\theta) = (0)|F|\sin(\theta) = 0 \]

Therefore, you don't have to worry about unknown forces acting at the point about which you are calculating the net torque.

In this problem, which of the following is the best point around which to calculate the torques?

ANSWER: the pivot at the wall
the point where the professor is standing
the center of mass of the beam
the end of the beam where the rope is attached
the center of the pulley

Once you have found all of the torques \( \tau_i \) (both magnitude and sign), add them up and set the sum equal to zero. This is the equilibrium condition:

\[ \sum_i \tau_i = \tau_{net} = 0 \]

Then, solve this equation for \( T \).

Hint A.2 Find the net torque
What is the net torque \( \tau_{net} \) on the beam around the pivot where the beam connects to the wall? Use \( T \) for the tension, \( W_p \) for the professor's weight, and \( W_b \) for the beam's weight.

Hint A.2.1 Find the magnitude of the normal force
What is the magnitude \( N \) of the normal force exerted by the professor on the beam? There are two key ideas needed to solve this. First, the normal force exerted by the professor on the beam has the same magnitude as and opposite direction from (i.e., it is the Newton's third law partner) the normal force \( F_N \) exerted by the beam on the professor. Second, the professor is in static equilibrium, and thus the net force on her must be zero.

Hint A.2.1.1 Forces on the professor
There are three forces acting on the professor: gravity (her weight), tension, and the normal force exerted by the beam. Gravity is directed downward, and the other two forces are directed upward. Thus, the equilibrium condition \( F_c = 0 \) can be solved to find the magnitude of the normal force in terms of given quantities.

Express your answer in terms of some or all of the variables \( W_p \), \( W_b \), and \( T \).

ANSWER: \[ N = W_p - T \]
**Hint A.2.2 Center of mass of the beam**
The weight of the beam acts at its center of mass. Since the beam is said to be uniform, the center of mass will be in the geometric center, 2.5 meters from each end.

Express your answer in terms of some or all of the variables \( w_b \), \( w_p \), and \( T \). Take torques that result in counterclockwise angular acceleration being positive.

**ANSWER:**

\[
\tau_{net} = \frac{5T}{\sqrt{2}} - 2T - 2w_p - \frac{5w_b}{2}
\]

Now set the net torque equal to zero and solve for \( T \). This will tell you the tension necessary for equilibrium, which is the tension in the rope, since we know that the professor/beam system is in equilibrium.

Express your answer in terms of \( w_b \) and \( w_p \).

**ANSWER:**

\[
T = \frac{2w_p + 2.5w_b}{\sqrt{2}/2}
\]

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**Part B**

If the professor's mass is 60 kilograms and the mass of the beam is 20 kilograms, what is the magnitude of the normal force exerted by the beam on the professor? Use 10 meters per second squared for the magnitude of the acceleration due to gravity.

**Hint B.1 How to approach the problem**

In Part A you found an expression for the magnitude of the normal force in the course of determining the net torque on the beam. Use that expression, together with the formula for tension that you found above, to determine the magnitude of the normal force in this situation.

Express your answer in newtons to two significant figures.

**ANSWER:**

\[
n = 290 \text{ N}
\]

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This applet shows all of the forces and how they change as the weight of the professor is varied.

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As an incentive to her students, the professor attaches a light basket to the center of mass of the beam and tells them that every student who gets an A on the next test gets to put a ball of mass 4 kilograms into the basket. There are enough students in the class that if a high percentage get A's, the weight of the basket will hoist the professor into the air.

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**Part C**

How many students have to get A's on the test to hoist the professor up into the air?

**Hint C.1 How to approach the problem**

You could simply add the weights of the balls as another torque in the problem. However, since the basket is located at the center of mass of the beam (where the weight of the beam is applied), the weight in the basket can be considered as an increase in the beam's weight \( w_b \).

Before you can calculate the weight in the basket that will cause the professor to be hoisted into the air, you have to understand why this should happen at all. Specifically, you need to figure out what condition will lead to her moving upward, instead of staying in equilibrium. Which of the statements below best describes the point at which equilibrium is lost and the professor begins moving upward?

**ANSWER:**

- The tension \( T \) exceeds the normal force \( N \).
- The tension \( T \) exceeds the weight of the professor \( w_p \).
- The weight of the beam \( w_b \) exceeds the weight of the professor \( w_p \).
- The weight of the beam \( w_b \) exceeds the tension \( T \).
- The center of mass of the system moves to the right so that it is no longer supported by the pulley.
- The change in weight of the beam causes the tension to be directed at an angle greater than 45 degrees.
The tension exceeding the weight violates the equilibrium condition on the professor: $F_{\text{net}} = 0$. If the tension is greater than the professor's weight, then this condition cannot possibly be met, and so she must be moving (that is, no longer in equilibrium). While you worked Part A, giving attention explicitly to the equilibrium condition of the beam, the value you used for the normal force was based on the equilibrium condition for the professor, which of course has to be met for the whole system to be in equilibrium.

Now that you know the condition that breaks the equilibrium, you can just set up the equation for that condition and solve it. In this case, set $w_{\text{prof}} = T$, using the expression for $T$ that you found above. Solve for the weight of the beam that satisfies this condition. For any $w_{\text{beam}}$ greater than that value, the professor will be lifted into the air, since the equilibrium conditions for the beam $T_{\text{net}} = 0$ and for the professor $F_{\text{net}} = 0$ can no longer be met.

Express your answer as an integer.

**ANSWER:** 17

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**Problem 12.38**

**Description:** The raised span of the drawbridge shown in the figure has its 11000-kg mass distributed uniformly over its 14-m length. (a) Find the force exerted by the drawbridge hinge as $F_{\text{vec}} = F_x \hat{i} + F_y \hat{j}$. Orts $\hat{i}$ and $\hat{j}$ are directed right and upward respectively.

The raised span of the drawbridge shown in the figure has its 11000-kg mass distributed uniformly over its 14-m length.

**Part A**

Find the force exerted by the drawbridge hinge as $F = F_x \hat{i} + F_y \hat{j}$. Orts $\hat{i}$ and $\hat{j}$ are directed right and upward respectively.

Express your answer using two significant figures.

**ANSWER:**

$F = 170 \hat{i} + 150 \hat{j}$ kN

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**Problem 12.33**

**Description:** The boom in the crane of the figure is free to pivot about point $P$ and is supported by the cable that joins halfway along its 18-m total length. The cable passes over a pulley and is anchored at the back of the crane. The boom has mass $m_1$ distributed along its length, and the mass hanging from the end of the boom is 2500 kg. The boom makes a 50° angle with the horizontal.

The boom in the crane of the figure is free to pivot about point $P$ and is supported by the cable that joins halfway along its 18-m total length. The cable passes over a pulley and is anchored at the back of the crane. The boom has mass 1800 distributed uniformly along its length, and the mass hanging from the end of the boom is 2500 kg. The boom makes a 50° angle with the horizontal.
Part A
What is the tension in the cable that supports the boom?

Express your answer using two significant figures.

\[ T = \frac{(m1 + 2m2)g}{\tan\left(\frac{50}{180}\right)} \text{ N} \]

Problem 12.45

Description: A 5.0-m-long ladder has mass \( m \) and is leaning against a frictionless wall, making a 66 degree angle with the horizontal. (a) If the coefficient of friction between the ladder and ground is 0.42, what is the mass of the heaviest person who can... A 5.0-m-long ladder has mass 12.0 kg and is leaning against a frictionless wall, making a 66 ° angle with the horizontal.

Part A
If the coefficient of friction between the ladder and ground is 0.42, what is the mass of the heaviest person who can safely ascend to the top of the ladder? (The center of mass of the ladder is at its center.)

Express your answer using two significant figures.

\[ m_{\text{max}} = \frac{m \left(0.5 - 0.42 \tan\left(\frac{50}{180}\right)\right)}{0.42 \tan\left(\frac{50}{180}\right) - 1} \text{ kg} \]

Problem 12.40

Description: A crane in a marble quarry is mounted on the rock walls of the quarry and is supporting a \( m_2 \) slab of marble as shown in the figure. The center of mass of the \( m_1 \) boom is located one-third of the way from the pivot end of its 15-m length, as shown. (a) ... A crane in a marble quarry is mounted on the rock walls of the quarry and is supporting a 2200 kg slab of marble as shown in the figure. The center of mass of the 850 kg boom is located one-third of the way from the pivot end of its 15-m length, as shown.

Part A
Find the tension in the horizontal cable that supports the boom.

\[ T = \frac{0.001 \left(m_2 + \frac{m_1}{3}\right)g}{\tan\left(\frac{50}{180}\right)} \text{ kN} \]

Problem 12.39
Description: Climbers attempting to cross a stream place a log against a vertical, frictionless ice cliff on the opposite side (see the figure). The log is inclined at 27 degrees, and its center of gravity is one-third of the way along its 6.3-m length. (a)...

Climbers attempting to cross a stream place a log against a vertical, frictionless ice cliff on the opposite side (see the figure). The log is inclined at 27 degrees, and its center of gravity is one-third of the way along its 6.3-m length.

Part A
If the coefficient of friction between the left end of the log and the ground is 0.92, what is the maximum mass for a climber and pack to cross without the log slipping?

Express your answer using two significant figures.

Answer:
\[ m_{\text{max}} = \frac{m \left( \tan \left( \frac{27}{180} \right) - \frac{1}{3} \right)}{1 - \mu \tan \left( \frac{27}{180} \right)} \text{ kg} \]

Problem 12.32

Description: A uniform ladder is leaning against a frictionless vertical wall, with which it makes a 15 degree angle. The coefficient of friction between ladder and ground is 0.26. (a) Can a person climb to the top of the ladder without it slipping? (b)...

A uniform 6.8 kg ladder is leaning against a frictionless vertical wall, with which it makes a 15 degree angle. The coefficient of friction between ladder and ground is 0.26.

Part A
Can a 65 kg person climb to the top of the ladder without it slipping?

Answer:
- yes
- no

Part B
If so, how massive a person would make the ladder slip?

Express your answer using two significant figures.

Answer:
\[ m_{\text{max}} = \frac{m_l \left( 0.26 - 0.5 \tan \left( \frac{15}{180} \right) \right)}{\tan \left( \frac{15}{180} \right) - 0.26} \text{ kg} \]

Score Summary:
Your score on this assignment is 0%.
You received 0 out of a possible total of 53 points.